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Exact evaluation of the nuclear form factor for new kinds of majoron emission in neutrinoless double beta decay *

C. Barbero^{a†}, J. M. Cline^b, F. Krmpotić^{a†} and D. Tadić^c

^a *Departamento de Física, Facultad de Ciencias Exactas
 Universidad Nacional de La Plata, C. C. 67, 1900 La Plata, Argentina.*

^b *McGill University, Montréal, Québec H3A 2T8, Canada.*

^c *Physics Department, University of Zagreb
 Bijenička c. 32-P.O.B. 162, 41000 Zagreb, Croatia.*

Abstract

We have developed a formalism, based on the Fourier-Bessel expansion, that facilitates the evaluation of matrix elements involving nucleon recoil operators, such as appear in several exotic forms of neutrinoless double beta decay ($\beta\beta_{0\nu}$). The method is illustrated by applying it to the “charged” majoron model, which is one of the few that can hope to produce an observable effect. From our numerical computations within the QRPA performed for ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{150}Nd nuclei, we test the validity of approximations made in earlier work to simplify the new matrix elements, showing that they are accurate to within 15%. Our new method is also suitable for computing other previously unevaluated $\beta\beta_{0\nu}$ nuclear matrix elements.

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[†]Fellow of the CONICET from Argentina.

Since the classic majoron model of Gelmini and Roncadelli [1] was excluded by the LEP measurement of the invisible width of the Z boson [2, 3], several new possibilities have been proposed for generating exotic neutrinoless double-beta decay with the emission of a massless majoron ($\beta\beta_M$). An interesting class of such models are those where the majoron is either a scalar [4] or a vector [5] boson that has lepton number -2 to balance that of the emitted electrons. These “charged majoron” models (CMM’s), so-called because the majoron carries the unbroken $U(1)$ charge of lepton number, are probably the only ones which have a chance of producing $\beta\beta_M$ at a rate which could be observed in the present generation of experimental searches [6, 7, 8].

From the point of view of nuclear physics, the CMM’s are interesting because their predicted rate of $\beta\beta_M$ depends on different matrix elements than in previously considered types of double beta decay, which had not been hitherto computed. In a previous publication [7] we made a first step toward computing the CMM matrix elements by neglecting a number of operators, an approximation which, while widely used in the literature, should be validated by more accurate computations. The aim of this letter is twofold. First, we elaborate a formalism, based on the Fourier-Bessel expansion introduced in ref. [9], that allows us to evaluate in a rather simple way the matrix elements of nuclear recoil operators, such as appear in the CMM. Second, to quantify the contributions of the operators neglected previously, we perform the corresponding numerical calculations in the QRPA for ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{150}Nd nuclei.

We begin by describing the matrix elements of interest and the approximations that were previously made. In the CMM of ref. [4], on which we will focus in this letter, the inverse half-life for $\beta\beta_M$ -decay is

$$[T(0_i^+ \rightarrow 0_f^+)]^{-1} = g_{CM}^2 \mathcal{G}_{CM} |\mathcal{M}_{CM}^+ - \mathcal{M}_{CM}^-|^2, \quad (1)$$

where g_{CM} is the effective majoron coupling defined in ref. [7],

$$\mathcal{G}_{CM} = \frac{(G_F \cos \theta_C)^4}{128\pi^7 \ln 2} \int (Q - \epsilon_1 - \epsilon_2)^3 \prod_{i=1}^2 k_i \epsilon_i F(\epsilon_i) d\epsilon_i, \quad (2)$$

is the kinematical factor in natural units [10] and

$$\mathcal{M}_{CM}^\pm = \frac{1}{4\pi} \langle 0_f^+ | \sum_{nm} \int k^2 dk v(k; M_\pm) \mathcal{O}_{CM}(k, \mathbf{r}_{nm}) | 0_i^+ \rangle, \quad (3)$$

with

$$O_{CM}(k, \mathbf{r}_{nm}) = i \int d\Omega_k \mathbf{k} \cdot \mathbf{Y}_{Rnm} e^{i\mathbf{k} \cdot \mathbf{r}_{nm}}, \quad (4)$$

are the nuclear form factors for charged majoron emission corresponding to the exchange of two neutrinos with masses M_{\pm} [7]. Here

$$v(k; M_{\pm}) = \frac{2}{\pi} \left\{ \frac{1}{M_{\pm}^2} \left[\frac{1}{\omega_{\pm}(\omega_{\pm} + \mu)} - \frac{1}{k(k + \mu)} \right] + \frac{1}{2} \frac{\partial}{\partial M_{\pm}^2} \frac{1}{\omega_{\pm}(\omega_{\pm} + \mu)} \right\}, \quad (5)$$

$\omega_{\pm} = (k^2 + M_{\pm}^2)^{\frac{1}{2}}$, and μ is the average excitation energy of the intermediate nuclear states. The operator

$$\mathbf{Y}_{Rnm} = i \left[g_A^2 (\boldsymbol{\sigma}_n C_m - C_n \boldsymbol{\sigma}_m) + i g_V g_A (\boldsymbol{\sigma}_n \times \mathbf{D}_m + \mathbf{D}_n \times \boldsymbol{\sigma}_m) + g_V^2 (\mathbf{D}_n - \mathbf{D}_m) \right], \quad (6)$$

with

$$\begin{aligned} C_n &= \frac{1}{2M} \boldsymbol{\sigma}_n \cdot (2\mathbf{p}_n + \mathbf{q}_n), \\ \mathbf{D}_n &= \frac{1}{2M} (2\mathbf{p}_n + \mathbf{q}_n + i f_W \boldsymbol{\sigma}_n \times \mathbf{q}_n), \end{aligned} \quad (7)$$

is the impulse non-relativistic approximation for the recoil term, obtained through the standard Foldy-Wouthuysen transformation [11]-[15]. M is the nucleon mass, $f_W = 4.7$ is the effective weak-magnetism coupling constant and $\mathbf{p}_n \equiv -i \boldsymbol{\nabla}_n^N$ and $\mathbf{q}_n \equiv -i \boldsymbol{\nabla}_n^L$ are, respectively, the nuclear and lepton operators acting at the point \mathbf{r}_n , with the understanding that $\boldsymbol{\nabla}_n^N$ operates only on the nucleon wave functions, while $\boldsymbol{\nabla}_n^L$ acts only on the exponential factor in eq. (4). Therefore one can make the replacements

$$\mathbf{q}_n \rightarrow \mathbf{k}, \quad \mathbf{q}_m \rightarrow -\mathbf{k}, \quad (8)$$

which yields¹

$$\begin{aligned} iM \mathbf{k} \cdot \mathbf{Y}_{Rnm} &= -g_A^2 [-(\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{k}) + (\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{p}_m) - (\boldsymbol{\sigma}_n \cdot \mathbf{p}_n)(\boldsymbol{\sigma}_m \cdot \mathbf{k})] \\ &\quad - g_V g_A [i\mathbf{k} \cdot (\boldsymbol{\sigma}_n \times \mathbf{p}_m + \mathbf{p}_n \times \boldsymbol{\sigma}_m) + f_W [(\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{k}) - k^2 (\boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m)] \\ &\quad - g_V^2 [(\mathbf{p}_n - \mathbf{p}_m) \cdot \mathbf{k} + k^2]. \end{aligned} \quad (9)$$

¹In eqs. (7) and (9) we have neglected the induced pseudoscalar interaction term. It is relatively small and can be easily embodied in eq. (9) by simply multiplying the operator $g_A^2 (\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{k})$ by the factor $(1 + h_P W_0/2)$, where $h_P = g_P/g_A \cong 2M/m_\pi^2 \sim 1/20$, and W_0 is $\beta\beta$ transition energy of the order of MeV.

In our previous work [7] we adopted the approximation made in another context by Doi *et al.*, [10]²

$$i\mathbf{k} \cdot \mathbf{Y}_{Rnm} \cong g_A \frac{k^2}{3M} \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m (2g_V f_W + g_A), \quad (10)$$

and which was also used by ref. [8]. Although a series of different justifications was given for this procedure, and we will expand upon them below, operationally it means to neglect in (9) all terms containing the nucleon momenta, as well as the term $g_V^2 k^2$, and to use the relation

$$(\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{k}) \cong \frac{1}{3} k^2 \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m. \quad (11)$$

However the nuclei momenta are necessarily of the same order as the momentum transfer k , and from this point of view the trustworthiness of the estimate (10) is questionable and it should be checked numerically. In the remainder of this letter we shall describe a method for so doing, and illustrate its use by computing for several nuclei the contributions to the matrix elements of $\mathbf{k} \cdot \mathbf{Y}_{Rnm}$ that were previously neglected.

To begin, we rewrite the operator $\mathcal{O}_{CM}(k, \mathbf{r}_{nm})$ in the form

$$\mathcal{O}_{CM}(k, \mathbf{r}_{nm}) = \mathcal{O}_{CM}^{AA}(k, \mathbf{r}_{nm}) + \mathcal{O}_{CM}^{AV}(k, \mathbf{r}_{nm}) + \mathcal{O}_{CM}^{VV}(k, \mathbf{r}_{nm}), \quad (12)$$

with

$$\begin{aligned} \mathcal{O}_{CM}^{AA}(k, \mathbf{r}_{nm}) &= \frac{g_A^2}{M} \int d\Omega_k e^{i\mathbf{k} \cdot \mathbf{r}_{nm}} [(\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{k}) - 2(\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{p}_m)], \\ \mathcal{O}_{CM}^{AV}(k, \mathbf{r}_{nm}) &= -\frac{g_A g_V}{M} \int d\Omega_k e^{i\mathbf{k} \cdot \mathbf{r}_{nm}} \left\{ i2\mathbf{k} \cdot \boldsymbol{\sigma}_n \times \mathbf{p}_m + f_W [(\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{k}) - k^2(\boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m)] \right\} \\ \mathcal{O}_{CM}^{VV}(k, \mathbf{r}_{nm}) &= -\frac{g_V^2}{M} \int d\Omega_k e^{i\mathbf{k} \cdot \mathbf{r}_{nm}} (2\mathbf{p}_n \cdot \mathbf{k} + k^2). \end{aligned} \quad (13)$$

In deriving the last result we used the fact that the terms containing \mathbf{p}_m contribute in the same way as the analogous terms containing \mathbf{p}_n , *i.e.*, the contribution of $\mathbf{k} \cdot (\boldsymbol{\sigma}_n \times \mathbf{p}_m)$ is equal to that of $-\mathbf{k} \cdot (\boldsymbol{\sigma}_m \times \mathbf{p}_n)$, *etc.*³ To obtain the corresponding matrix elements

$$\mathcal{M}_{CM}^{x\pm} = \frac{1}{4\pi} \langle 0_f^+ | \sum_{nm} \int k^2 dk v(k; M_{\pm}) \mathcal{O}_{CM}^x(k, \mathbf{r}_{nm}) | 0_i^+ \rangle, \quad (14)$$

²Our relative signs differ in this equation and in eq. (6) from the corresponding ones of ref. [10] because of a sign error in the latter.

³This can be easily shown from the multipole expansion that follows.

where $|0_i^+\rangle$ and $|0_f^+\rangle$ are the initial and final nuclear states, respectively, and x stands for AA , AV and VV , we do the following. First, we perform the multipole expansion in (13) and integrate over the angular coordinates $d\Omega_k$. Second, we use the angular momenta algebra and introduce a complete set of intermediate states $|J_\alpha^\pi\rangle$. In this way we get

$$\begin{aligned}\mathcal{M}_{CM}^{AA\pm} &= \frac{4\pi g_A^2}{M} \int dk k^3 v(k; M_\pm) \sum_{LL'J_\alpha^\pi} i^{L+L'} \\ &\times \left\{ 2\delta_{LL'} i^{L+J} (-1)^J \hat{L} \hat{J}^{-1} (L1|J) \langle 0_f^+ | \mathbb{T}_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) | J_\alpha^\pi \rangle \langle J_\alpha^\pi | \mathbb{Y}_J(k, \mathbf{r}, \boldsymbol{\sigma} \cdot \mathbf{p}) | 0_i^+ \rangle \right. \\ &- \left. k \hat{L} \hat{L}' \hat{J}^{-2} (L1|J) (L'1|J) \langle 0_f^+ | \mathbb{T}_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) | J_\alpha^\pi \rangle \langle J_\alpha^\pi | \mathbb{T}_{L'J}(k, \mathbf{r}, \boldsymbol{\sigma}) | 0_i^+ \rangle \right\},\end{aligned}\tag{15}$$

$$\begin{aligned}\mathcal{M}_{CM}^{AV\pm} &= \frac{4\pi g_A g_V}{M} \int dk k^3 v(k; M_\pm) \sum_{LL'J_\alpha^\pi} i^{L+L'} \\ &\times \left\{ -f_w k \delta_{LL'} \langle 0_f^+ | \mathbb{T}_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) | J_\alpha^\pi \rangle \langle J_\alpha^\pi | \mathbb{T}_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) | 0_i^+ \rangle \right. \\ &+ f_w k \hat{L} \hat{L}' \hat{J}^{-2} (L1|J) (L'1|J) \langle 0_f^+ | \mathbb{T}_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) | J_\alpha^\pi \rangle \langle J_\alpha^\pi | \mathbb{T}_{L'J}(k, \mathbf{r}, \boldsymbol{\sigma}) | 0_i^+ \rangle \\ &- \left. 2(-1)^{L+J} \sqrt{6} \hat{L} \begin{Bmatrix} L & J & 1 \\ 1 & 1 & L' \end{Bmatrix} (L1|L') \langle 0_f^+ | \mathbb{T}_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) | J_\alpha^\pi \rangle \langle J_\alpha^\pi | \mathbb{T}_{L'J}(k, \mathbf{r}, \mathbf{p}) | 0_i^+ \rangle \right\},\end{aligned}\tag{16}$$

$$\begin{aligned}\mathcal{M}_{CM}^{VV\pm} &= \frac{4\pi g_V^2}{M} \int dk k^3 v(k; M_\pm) \sum_{LJ_\alpha^\pi} \\ &\times \left\{ -k \delta_{LJ} (-1)^J \langle 0_f^+ | \mathbb{Y}_J(k, \mathbf{r}, 1) | J_\alpha^\pi \rangle \langle J_\alpha^\pi | \mathbb{Y}_J(k, \mathbf{r}, 1) | 0_i^+ \rangle \right. \\ &- \left. 2i^{L+J} \hat{L} \hat{J}^{-1} (L1|J) \langle 0_f^+ | \mathbb{Y}_J(k, \mathbf{r}, 1) | J_\alpha^\pi \rangle \langle J_\alpha^\pi | \mathbb{T}_{LJ}(k, \mathbf{r}, \mathbf{p}) | 0_i^+ \rangle \right\}.\end{aligned}\tag{17}$$

where $\hat{J} \equiv \sqrt{2J+1}$, $(L1|J)$ is a short notation for the Clebsh-Gordon coefficient $(L010|J0)$ and

$$\begin{aligned}\mathbb{Y}_{LM}(k, \mathbf{r}, 1) &= j_L(kr) Y_{LM}(\hat{\mathbf{r}}), \\ \mathbb{Y}_{LM}(k, \mathbf{r}, \boldsymbol{\sigma} \cdot \mathbf{p}) &= j_L(kr) Y_{LM}(\hat{\mathbf{r}}) (\boldsymbol{\sigma} \cdot \mathbf{p}), \\ \mathbb{T}_{LJM}(k, \mathbf{r}, \boldsymbol{\sigma}) &= j_L(kr) (\boldsymbol{\sigma} \otimes Y_L(\hat{\mathbf{r}}))_{JM}, \\ \mathbb{T}_{LJM}(k, \mathbf{r}, \mathbf{p}) &= j_L(kr) (\mathbf{p} \otimes Y_L(\hat{\mathbf{r}}))_{JM},\end{aligned}\tag{18}$$

are spherical one-body operators [11], the reduced matrix elements of which appear in eqs. (15-17). The explicit dependence on the energies $\omega_{J_\alpha^\pi}$ of the intermediate states $|J_\alpha^\pi\rangle$ in the latter equations can be restored, if desired, by replacing $\mu \rightarrow \omega_{J_\alpha^\pi}$ in eq. (5). The total matrix element is clearly

$$\mathcal{M}_{CM}^\pm = \mathcal{M}_{CM}^{AA^\pm} + \mathcal{M}_{CM}^{AV^\pm} + \mathcal{M}_{CM}^{VV^\pm}, \quad (19)$$

and in the approximation of eq. (10) this simplifies to

$$\begin{aligned} \mathcal{M}_{CM}^\pm &\cong \frac{4\pi g_A(2g_V f_w + g_A)}{3M} \int dk k^4 v(k; M_\pm) \sum_{LJ_\alpha^\pi} (-1)^{L+1} \\ &\times \langle 0_f^+ || \mathcal{T}_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) || J_\alpha^\pi \rangle \langle J_\alpha^\pi || \mathcal{T}_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) || 0_i^+ \rangle. \end{aligned} \quad (20)$$

In what follows we shall compute both the the full matrix element and this approximation in order to compare the two.

The results derived so far are valid for any nuclear model; it remains only to evaluate the reduced matrix elements $\langle 0_f^+ || \mathcal{O}_{LJ}(k, \mathbf{r}) || J_\alpha^\pi \rangle$ and $\langle J_\alpha^\pi || \mathcal{O}'_{LJ}(k, \mathbf{r}) || 0_i^+ \rangle$ of the operators $\mathcal{O}_{LJ}(k, \mathbf{r})$ and $\mathcal{O}'_{LJ}(k, \mathbf{r})$ listed in (18). Within the QRPA formulation, after solving the BCS equations for the intermediate nucleus [16], the transition matrix elements become

$$\begin{aligned} \langle 0_f^+ || \mathcal{O}_{LJ}(k, \mathbf{r}) || J_\alpha^\pi \rangle &= - \sum_{pn} \langle p || \mathcal{O}_{LJ}(k, \mathbf{r}) || n \rangle [v_p u_n X_{pn;\alpha J} + u_p v_n Y_{pn;\alpha J}], \\ \langle J_\alpha^\pi || \mathcal{O}'_{LJ}(k, \mathbf{r}) || 0_i^+ \rangle &= - \sum_{pn} \langle p || \mathcal{O}'_{LJ}(k, \mathbf{r}) || n \rangle [u_p v_n X_{pn;\alpha J} + v_p u_n Y_{pn;\alpha J}], \end{aligned} \quad (21)$$

and the BCS approximation results from the substitution:

$$\begin{aligned} &\sum_\alpha \langle 0_f^+ || \mathcal{O}_{LJ}(k, \mathbf{r}) || J_\alpha^\pi \rangle \langle J_\alpha^\pi || \mathcal{O}'_{LJ}(k, \mathbf{r}) || 0_i^+ \rangle \\ &\rightarrow \sum_{pn} \langle p || \mathcal{O}_{LJ}(k, \mathbf{r}) || n \rangle \langle p || \mathcal{O}'_{LJ}(k, \mathbf{r}) || n \rangle u_p v_n v_p u_n. \end{aligned} \quad (22)$$

For harmonic oscillator radial wave functions, the reduced single-particle pn form factors are [11, 17]

$$\begin{aligned} \langle p || Y_J(k, \mathbf{r}, 1) || n \rangle &= (4\pi)^{-\frac{1}{2}} W(pn; J0J) R^0(pn; J, k), \\ \langle p || T_{LJ}(k, \mathbf{r}, \boldsymbol{\sigma}) || n \rangle &= (4\pi)^{-\frac{1}{2}} W(pn; L, 1, J) R^0(pn; L, k), \\ \langle p || T_{LJ}(k, \mathbf{r}, \mathbf{p}) || n \rangle &= (4\pi)^{-\frac{1}{2}} [W_1^{(-)}(pn; L, J) R^{(-)}(pn; L, k) \end{aligned}$$

$$\begin{aligned}
& + W_1^{(+)}(pn; L, J)R^{(+)}(pn; L, k) \Big], \\
\langle p || Y_J(k, \mathbf{r}, \boldsymbol{\sigma} \cdot \mathbf{p}) || n \rangle & = (4\pi)^{-\frac{1}{2}} \Big[W_2^{(-)}(pn; J)R^{(-)}(pn; J, k) \\
& + W_2^{(+)}(pn; J)R^{(+)}(pn; J, k) \Big], \tag{23}
\end{aligned}$$

with the angular parts⁴

$$\begin{aligned}
W(pn; L, S, J) & = \sqrt{2} \hat{S} \hat{J} \hat{L} \hat{l}_n \hat{j}_n \hat{j}_p (l_n L | l_p) \begin{Bmatrix} l_p & \frac{1}{2} & j_p \\ L & S & J \\ l_n & \frac{1}{2} & j_n \end{Bmatrix}, \\
W_1^{(\pm)}(pn; L, J) & = \mp i (-1)^{l_p + j_n + J + \frac{1}{2}} \hat{J} \hat{L} \hat{l}_p \hat{j}_p \hat{j}_n (l_n + \frac{1}{2} \mp \frac{1}{2})^{\frac{1}{2}} (l_p L | l_n \mp 1) \\
& \times \begin{Bmatrix} l_p & j_p & \frac{1}{2} \\ j_n & l_n & J \end{Bmatrix} \begin{Bmatrix} L & J & 1 \\ l_n & l_n \mp 1 & l_p \end{Bmatrix}, \\
W_2^{(\pm)}(pn; J) & = \mp i (-1)^{l_n + j_n + J + \frac{1}{2}} \sqrt{6} \hat{J} \hat{l}_p \hat{j}_p \hat{j}_n (l_n + \frac{1}{2} \mp \frac{1}{2})^{\frac{1}{2}} (l_p J | l_n \mp 1) \\
& \times \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ j_n & l_n & l_n \mp 1 \end{Bmatrix} \begin{Bmatrix} l_n \mp 1 & j_n & \frac{1}{2} \\ j_p & l_p & J \end{Bmatrix}, \tag{24}
\end{aligned}$$

and the radial parts

$$\begin{aligned}
R^0(pn; L, k) & \equiv R^L(k; l_p, n_p, l_n, n_n) = \int_0^\infty u_{n_p, l_p}(r) u_{n_n, l_n}(r) j_L(kr) r^2 dr \\
R^{(\pm)}(pn; L, k) & = \pm \left(\frac{\nu}{2} \right)^{\frac{1}{2}} \left\{ (2l_n + 2n_n + 2 \mp 1)^{\frac{1}{2}} R^L(k; l_p, n_p, l_n \mp 1, n_n) \right. \\
& \left. + (2n_n + 1 \pm 1)^{\frac{1}{2}} R^L(k; l_p, n_p, l_n \mp 1, n_n \pm 1) \right\}, \tag{25}
\end{aligned}$$

where $\nu = M\omega/\hbar$ is the oscillator parameter. We use here $g_A = g_V$, a value almost universally adopted for the effective axial vector coupling constant, both in single and double beta nuclear decays. As in our previous work [7]⁵ we made the numerical calculations in the simplifying limit of $M_+ = \infty$, which implies $\mathcal{M}_{CM}^+ = 0$, and $M_- = 100$ MeV, and we followed the QRPA procedure described in refs. [16, 18].⁶ For the discussion that follows

⁴We use here the angular momentum coupling $|(\frac{1}{2}, l) j \rangle$.

⁵There was a misprint in ref. [7] requiring the following replacement:

$$\mathcal{M}_{CM}^\pm = \sum_{LSJ^\pi} m(M_\pm; L, S, J^\pi) \rightarrow \mathcal{M}_{CM}^\pm = \sum_{LJ^\pi} m(M_\pm; L, S = 1, J^\pi).$$

⁶To assess the reliability of the nuclear model employed here, it is worthwhile to compare the recently

we will only need the parameters s and t , defined as the ratios between the $T = 1, S = 0$ and $T = 0, S = 1$ coupling constants in the particle-particle (PP) channels and the pairing force constants, *i.e.*, $s = 2v_s^{pp}/[v_s^{pair}(p) + v_s^{pair}(n)]$ and $t = 2v_t^{pp}/[v_s^{pair}(p) + v_s^{pair}(n)]$. (For a value of $s \cong 1$ the isospin symmetry is restored within the QRPA.)

Table 1 gives the contributions of the individual matrix elements in (12) and (13) and their combinations for the charged majoron $\beta\beta_M$ decay in ^{76}Ge . Three different results are shown: i) the unperturbed or BCS values (second column), ii) the QRPA calculations when only the particle-hole interaction is considered, *i.e.*, with $s = t = 0$ (third column) and iii) the full QRPA calculations with $s = 1$ and $t = 1.25$ (fourth column). From the results for the Gamow-Teller like operators $k^2\boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m/M$ and $\boldsymbol{\sigma}_n \cdot \mathbf{k}\boldsymbol{\sigma}_m \cdot \mathbf{k}/M$ we see that the approximation (11) is always quite good. This is also true for the individual contributions, coming both from the natural parity ($\pi = (-)^J$) and the unnatural parity ($\pi = (-)^{J+1}$) virtual states J^π . The last two rows show that the approximation (10) is also quite reasonable. This is clearly because the contributions coming from the velocity dependent operators, as well as from the Fermi-like operator k^2/M , are relatively small. It should be noticed that the operators $-2(\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{p}_m)/M$ and $-2i\mathbf{k} \cdot \boldsymbol{\sigma}_n \times \mathbf{p}_m/M$ ($-2\mathbf{p}_n \cdot \mathbf{k}/M$ and $-k^2/M$) are forbidden by the L and J selection rules for the natural (unnatural) parity intermediate states in eqs. (15), (16) and (17). Moreover, while all intermediate states contribute coherently for the velocity independent operators, the same is not true of the velocity dependent operators. The latter are also more sensitive to the PP correlations than the former, to the extent that their contributions pass through zero for $t \leq 1.25$. Table 2 shows the comparison of the exact QRPA results from eq. (19), for several $\beta\beta$ decaying nuclei, with those resulting from the approximation (20). We have used here $s = 1$ and $t = t_{sym}$ as discussed in ref. [18], *i.e.*, $t = 1.25, 1.30, 1.50, 1.40$ and 1.85 for ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{150}Nd , respectively. In all cases the exact matrix elements are larger in magnitude than the approximate ones by $\leq 15\%$.

We would also like to point out that our formulation is applicable to matrix elements

reported result $T_{1/2}^{2\nu} = (4.3_{-1.1}^{+2.4}[\text{stat.}] \pm 1.4[\text{syst.}]) \times 10^{19}$ y for the $\beta\beta_{2\nu}$ half-life measurement in ^{48}Ca , with the predicted value (using the same nuclear structure technique) $T_{1/2}^{2\nu} = 2.8 \times 10^{19}$ y [18]. When the calculation was done only the lower limit of the 2ν decay half-life, $T_{1/2}^{2\nu} \geq 3.6 \times 10^{19}$ y, was known experimentally. On the other hand, the nuclear shell model [19] restricts the corresponding half-life from above, $T_{1/2}^{2\nu} \leq 10^{20}$ y.

that appear in the usual $\beta\beta_{0\nu}$ -decay, as well some supersymmetric contributions. For the former, the neutrino mass term has been evaluated within this framework in refs. [9, 18]. In addition, the recoil matrix element [20]-[23]

$$M_R'^{(0\nu)} = \left\langle -i \int \frac{d\mathbf{k}}{(2\pi)^2} \mathbf{k} \cdot (\boldsymbol{\sigma}_n \times \mathbf{D}_m + \mathbf{D}_n \times \boldsymbol{\sigma}_m) \frac{e^{i\mathbf{k} \cdot \mathbf{r}_{nm}}}{k(k + \mu)} \right\rangle, \quad (26)$$

which contributes to neutrinoless $\beta\beta$ decay in models where the right-handed ($V + A$) exists along with the usual left-handed ($V - A$) one, can be easily calculated from eq. (16) by making the substitution

$$g_A g_V v(k; M_{\pm}) \rightarrow \frac{1}{\pi} \frac{1}{k(k + \mu)}. \quad (27)$$

(The finite nucleon size effect and the short-range two-nucleon correlations can also be incorporated in a simple way [18].⁷) Furthermore in supersymmetric models [24] a matrix element arises with a neutrino potential of the form

$$h_R(r) \sim \frac{2}{\pi} \int dk \frac{k^4}{\omega(\omega + \mu)} j_0(kr) = \int \frac{d\mathbf{k}}{2\pi^2} \frac{k^2}{\omega(\omega + \mu)} e^{i\mathbf{k} \cdot \mathbf{r}}, \quad (28)$$

whose extra powers of k^2 relative to the usual neutrino potential show that this is similar to the recoil-like terms we evaluated above, making this matrix element also amenable to calculation by our method.

In summary, we have developed a formalism especially suited for computing nuclear form factors that contain nuclear recoil operators, regardless of the nuclear model, and we applied it to the matrix elements for charged majoron emission in the QRPA. We thereby found that the approximations used in our previous paper [7] to simplify the nuclear form factor (6) work quite well, to within a relative error of 15%.

⁷ So far the matrix element $M_R'^{(0\nu)}$ has been handled as a two-body operator, which leads to rather complicated analytical expressions (see eqs. (2.20) to (2.27) in ref. [21] and eqs. (3.65) to (3.68) in ref. [23]).

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Table 1: Contributions of the individual matrix elements and their combinations for the charged majoron decay in ^{76}Ge .

Operator	BCS	ph-QRPA	full-QRPA
$k^2 \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m / M$	0.200	0.165	0.112
$\boldsymbol{\sigma}_n \cdot \mathbf{k} \boldsymbol{\sigma}_m \cdot \mathbf{k} / M$	0.067	0.055	0.037
$-k^2 / M$	0.067	0.049	0.035
$-2(\boldsymbol{\sigma}_n \cdot \mathbf{k})(\boldsymbol{\sigma}_m \cdot \mathbf{p}_m) / M$	-0.026	-0.014	0.007
$-2i\mathbf{k} \cdot \boldsymbol{\sigma}_n \times \mathbf{p}_m / M$	0.016	0.014	-0.004
$-2\mathbf{p}_n \cdot \mathbf{k} / M$	0.006	0.006	0.006
$-g_A^2 \mathbf{k} \cdot (\boldsymbol{\sigma}_n C_m - C_n \boldsymbol{\sigma}_m)$	0.041	0.041	0.044
$-ig_V g_A \mathbf{k} \cdot (\boldsymbol{\sigma}_n \times \mathbf{D}_m + \mathbf{D}_n \times \boldsymbol{\sigma}_m)$	0.640	0.530	0.348
$-g_V^2 \mathbf{k} \cdot (\mathbf{D}_n - \mathbf{D}_m)$	0.073	0.054	0.041
$i\mathbf{k} \cdot \mathbf{Y}_{Rnm}$	0.754	0.624	0.433
$g_A k^2 \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m (2g_V f_W + g_A) / 3M$	0.694	0.571	0.387

Table 2: Results for the exact and approximated matrix elements, given eqs. (19) and (20), respectively. Both the particle-hole and the particle-particle channels have been considered in the QRPA calculations.

Operator	^{76}Ge	^{82}Se	^{100}Mo	^{128}Te	^{150}Nd
$i\mathbf{k} \cdot \mathbf{Y}_{Rnm}$	0.433	0.437	0.444	0.412	0.345
$g_A k^2 \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m (2g_V f_W + g_A) / 3M$	0.387	0.388	0.380	0.359	0.288